Fermi National Accelerator Laboratory

FERMILAB-Pub -85/91-T
June, 1985

Passive Correction of Orbit Distortion in Very Large Proton Storage Rings

JONATHAN F. SCHONFELD*
Fermi National Accelerator Laboratory
Batavia, Illinois 60510

ABSTRACT

We propose and discuss a simple magnetic technique for suppressing nonlinear orbit distortion in proton storage rings. This work was motivated by design needs of the proposed Superconducting Super Collider (SSC). (PACS Numbers 03.20.+i, 29.20.-c, 29.20.Dh,29.20.Lq,41.70.+t, 41.80.-y,46.10.+z.)

*Address after July 8, 1985: MIT Lincoln Laboratory, P. O. Box 73, Lexington MA 02173.

Operated by Universities Research Association Inc. under contract with the United States Department of Energy

Computer simulation and analytical theory play influential but problematical roles in the design of very high energy storage rings, such as the proposed Superconducting Super Collider (SSC)[1]. Simulation can tell us when a design cannot work, but it cannot at present tell us what changes to make. Theory gives us simple formulae for particle orbits, but so far only in approximations that are generally mistrusted.

Serious work is in progress to improve these diagnostic and predictive tools,[2,3] prompted by early discouraging single-beam simulations[4] of the SSC Reference Design.[1] In this letter we propose and explore the first example of a third, intermediate approach: a scheme for correction magnets, suggested by perturbative analytic theory, to be used when computer simulation indicates trouble, and to be tested by further computer simulation. A more detailed treatment of these considerations is in preparation.[5]

Our idea is based on a simple formulation of first order perturbation theory due to Collins.[6] Betatron oscillation (transverse to some ideal design orbit) is written in the conventional form[7]

$$x = [2I_x\beta_x(s)]^{1/2}\cos\theta_x$$
$$y = [2I_y\beta_y(s)]^{1/2}\cos\theta_y \quad . \tag{1}$$

The parameter s is distance along the beam pipe. In zeroth order, the amplitudes I_x and I_y are constants. Collins writes the first order corrections to I_x and I_y as sums of terms of the form

$$I_x^{|n|/2} \; I_y^{|m|/2} \; D(s) \; \exp i(n\omega_x + m\omega_y)s \; \exp i\phi \quad , \tag{2}$$

where the ω's are unperturbed frequencies, n and m are integers, ϕ is an arbitrary fixed phase, and where the complex periodic functions generically written as D are called "distortion functions." Collins argues (this is not yet a theorem) that a well-behaved storage ring has small D's. A distortion functions is especially "user-friendly" because it satisfies the simple equation

$$[\frac{d}{ds} + i(n\omega_x + m\omega_y)]D = R(s), \tag{3}$$

where the source R measures some magnetic multipole strength other than dipole and normal quadrupole. The low-order D's include two skew quadrupole distortion functions, and five normal and five skew sextupole distortion functions.

Let N be the total number of dipole (bending) magnets (the most important sources of unwanted higher multipoles) in the ring. If these magnets have random and uncorrelated error multipoles with zero mean, then, from (3), the mean square magnitude of any particular distortion function is roughly proportional to N (ignoring variation in the beta functions). Our idea is to decrease distortion functions by partitioning the accelerator into M sectors, placing correction magnets in each sector so that the distortion functions in any sector receive contributions only from the magnets <u>in that sector</u>. As a result mean square distortion functions become roughly proportional to N/M instead of to N; i.e., we expect reduction by a factor of 1/M, times some function ξ of position that must be determined numerically.

In particular: Within each sector, for each complex distortion function that we want to suppress, we need two real correction magnets of appropriate multipolarity. (Precise placement of corrections is discussed below). Outside the sector in question, each of these corrections, and the in-sector multipoles that they are to compensate for, contributes only a constant times exp i $(n\omega_x + m\omega_y)s$ to a given D, by (3). The condition that all these constants add to zero gives a linear constraint on the correction strengths, with equal numbers of equations and unknowns.

To test this idea, one computes ξ for a given type of distortion and asks if ξ/M is small for some reasonable value of M. If it is, then one asks whether diminished distortion functions really lead to better behaved particle orbits.

We have applied this scheme to the five normal sextupole distortion functions of a (slightly modified) model storage ring for SSC magnet design evaluation that Norman Gelfand[4] developed for use with the orbit-tracking program TEVLAT.[8] There are 480 standard cells, as shown in Figure 1. The circumference is roughly 102 km, the vertical and horizontal tunes are close to 80 1/6, and therefore betatron phase advances per cell are very close to 60°. The mean square random normal sextupole strength localized in the centers of the model's 960 dipoles is $\ell\ (\partial^2 B_y/\partial x^2)/B\rho \cong 1.6$, where ℓ is the (infinitesimal) sextupole length, and $B\rho$ is the magnetic rigidity of a 20 TeV proton.

Each correction sector contains N/2M cells, and twelve correction sextupoles (the extra two serve to cancel induced chromaticity). For simplicity, we imagine each sector beginning with several cells in which corrections are placed in a repeating pattern, followed by a long string of cells with no corrections at all. It turns out[5] that when the phase advances per cell are close to 60^o, the linear conditions on the correction strengths are almost singular unless we put at least four correction magnets in each of the lead cells. We settled on placing a zero-length normal sextupole correction at the centers of each of the F and D quadrupoles and of the two bends in each of the first three cells in each sector. Of course other configurations are possible.

A check of the 1/M scaling law is shown in Figures 2 and 3. With this arrangement, the <u>maximum</u> value of ξ ranges from about 10 to about 40, depending on the particular distortion function, so that all sextupole distortion functions are significantly reduced everywhere only for M much greater than about 40 which leads to sectors so small that one becomes concerned about the cost and space needs of the correction system. We have therefore not submitted such a corrected lattice to an orbit tracking program. However, one can show[5] that away from cells with correction elements, the factor ξ varies roughly from no more than 4M/N at the end of a sector, to no more than four just after the last correction magnet. So, such a correction scheme might permit one to decrease magnet aperture everywhere except in the small fraction of the ring containing correction magnets.

Ideally, one would like to correct orbital problems <u>actively</u>, by using the results of orbit tracking programs--or even orbit measurements on the real storage ring--directly to determine correction strengths. In any case, perhaps one can enhance correction schemes by shuffling magnets.[9]

I have sought to publish these considerations in Physical Review Letters because most high energy physicists--to say nothing of our colleagues in other areas--are largely unaware of the technical problems presented by the SSC. Some of these hard problems are not receiving the broad and interdisciplinary attention that they deserve.

This work could not have been done without the generous and patient assistance and tutelage of Norman Gelfand. I am also grateful to Don Edwards, Leo Michelotti, Al Russel, and Roy Thatcher for helpful conversations. This research program grew out of talks with Christoph Leemann at the SSC Dynamic Aperture Workshop at LBL in November 1984. Finally, I am grateful to Phyllis and Sam Gorenstein for their hospitality while this paper was drafted.

REFERENCES

1. "Superconducting Super Collider," designs study for U. S. Department of Energy, May 8, 1984.

2. "SSC Aperture Workshop Summary, November 1984," SSC Report SSC-TR-2001.

3. L. Michelotti, Particle Accelerators 16, 233 (1985); "Deprit's Algorithm, Green's Functions, and Multipole Perturbation Theory," FERMILAB-Pub-85/63, presented at Workshop on Orbital Dynamics and Applications to Accelerators, Berkeley, Calif., March 7-12, 1985, submitted to Particle Accelerators.

4. G. F. Dell (Brookhaven), unpublished; N. Gelfand (Fermilab), unpublished; B. Leemann (LBL), unpublished.

5. J. F. Schonfeld, "A New Approach to Dynamic Aperture Problems," FERMILAB-Pub-85/112-T to be submitted to Particle Accelerators.

6. T. L. Collins, "Distortion Functions," FERMILAB-Pub-84/114, October 23, 1984. For further developments, see A. J. Dragt, "Nonlinear Lattice Functions," University of Maryland Physics Publication 85-004, July 1984; and K. Y. Ng, "Derivation of

Collins' Formulas for Beam-Shape Distortion Due to Sextupoles Using Hamiltonian Method," Fermilab report TM-1281, October 1984.

7. E. D. Courant and H. S. Snyder, Ann. Phys. (N. Y.) 3, 1 (1958).

8. A. D. Russel, "TEVLAT: A New Program for Computing Lattice Functions for the Energy Saver/Doubler," Fermilab report UPC No. 124, March 14, 1980.

9. R. L. Gluckstern and S. Ohnuma, "Reduction of Sextupole Distortion by Shuffling Magnets in Small Groups," Fermilab report TM-1312, to be published in Proceedings of the 1985 Particle Accelerator Conference, Vancouver, Canada, May 13-16, 1985.

FIGURE CAPTIONS

1. Standard cell in our version of N. Gelfand's SSC model. All lengths are approximate. Horizontally focussing (F) and defocusing (D) quadrupoles have strength ~ .04 M^{-2}. Each dipole has bend angle ~ 6.5 mrad.

2. Ratio of maximum square modulus of a sextupole distortion function (d_{10}, see Ref. 5) with corrections to without, as function of number M of correction sectors, by numerical calculation (squares) and by least-squares fit (ignoring M=1, see Ref. 5) to const./M.

3. Continuation of Fig. 2.

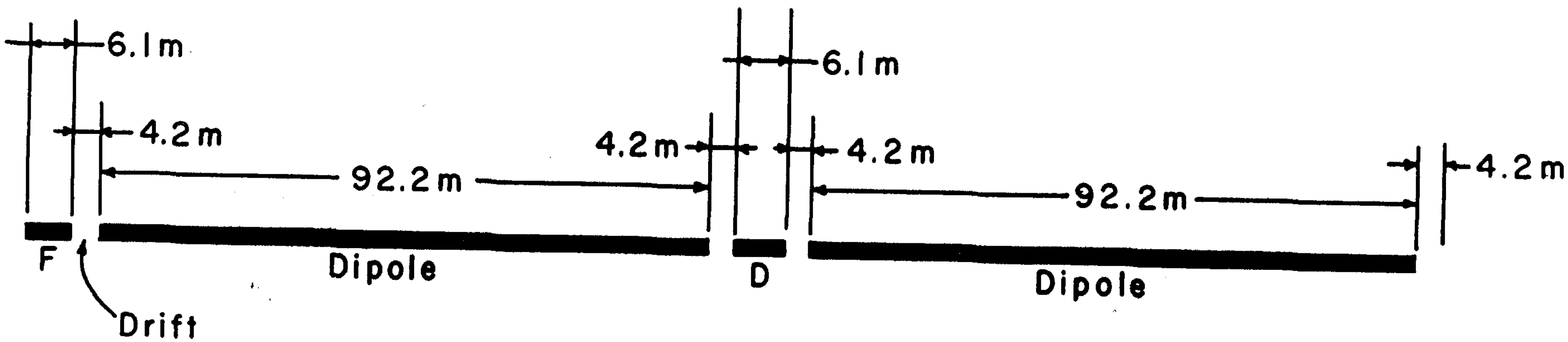

Figure 1

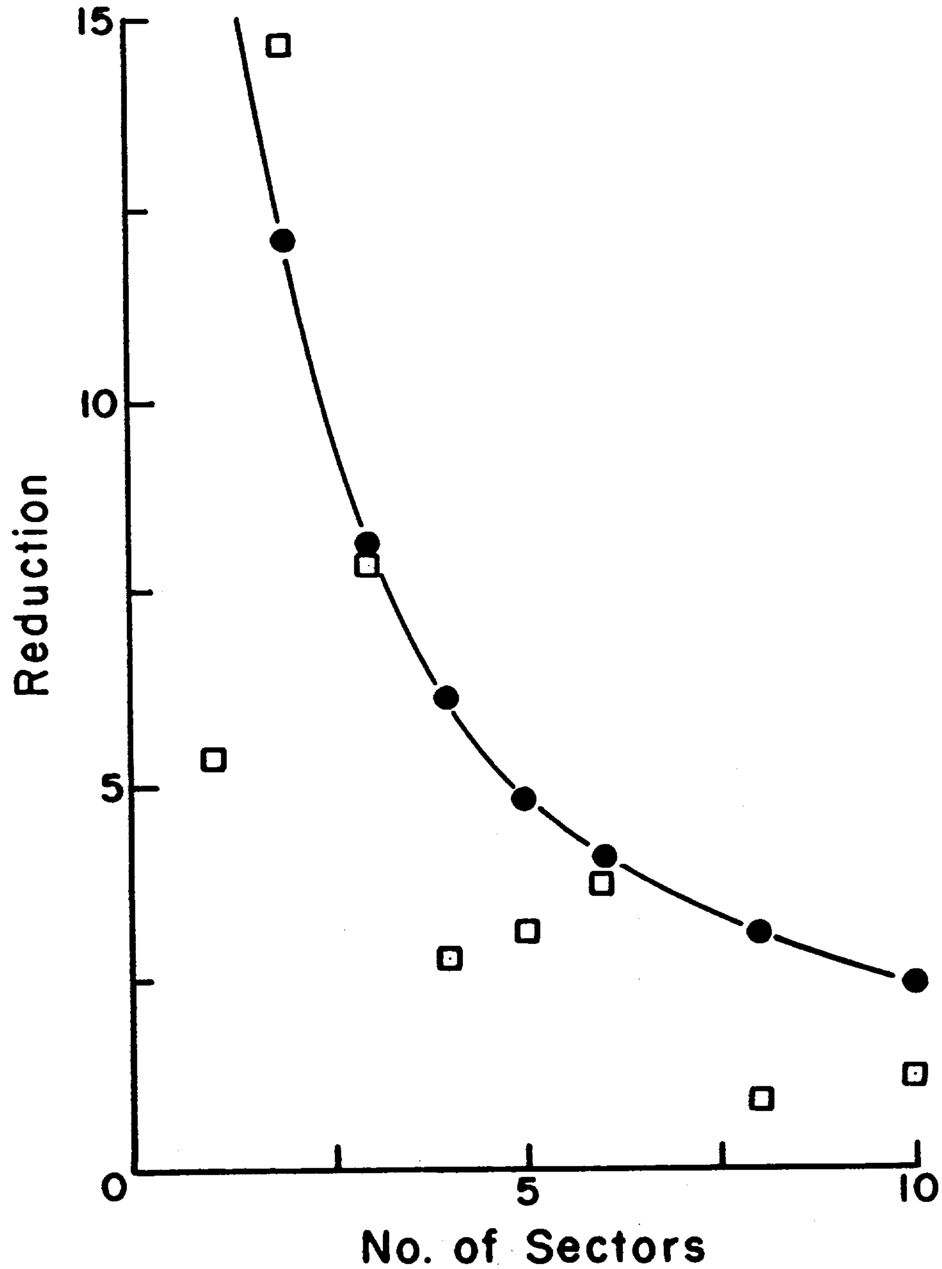

Figure 2

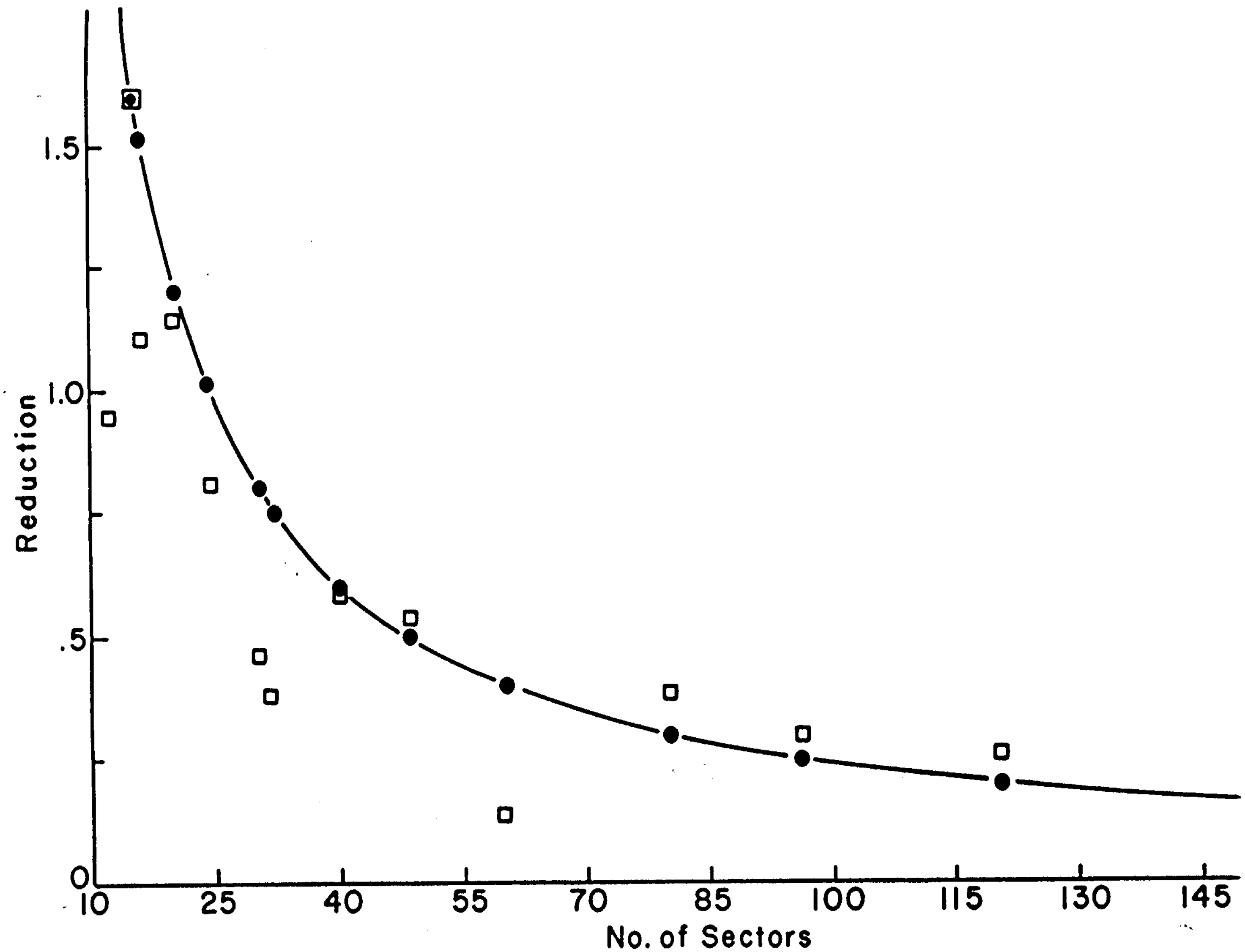
1.5
1.0
.5
0
Reduction
10
25
40
55
70
85
100
115
130
145
No. of Sectors

Figure 3